Exploring Position and Momentum in the Quantum World
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**Purpose**
This lesson is intended as a supplement to a lesson on relative position and velocity. It provides students with an opportunity to discuss frames of reference and connect that to how some scientists are probing the limits of Heisenberg’s uncertainty principle.

**Audience**
This lesson was designed for use in an introductory high school physics course, including AP Physics 1.

**Lesson Objectives**
Upon completion of this lesson, students will be able to:
- determine the relative position of two objects on an xy-coordinate plane.
- determine the velocity of an object using different frames of reference.
- compare and contrast finding relative position and velocity to how scientists could use negative mass particles to determine the exact position of positive mass particles.

**Key Words**
classical physics, Heisenberg’s uncertainty principle, quantum physics

**Big Question**
This lesson plan addresses the Big Question, “What does it mean to observe?”

**Standard Alignments**
- **Science and Engineering Practices**
  - SP 5. Using mathematics and computational thinking.
- **AP Physics 1 Standards**
  - Essential Knowledge 3.A.1. An observer in a particular reference frame can describe the motion of an object using such quantities as position, displacement, distance, velocity, speed, and acceleration.
  - Learning Objective 3.A.1.1. The student is able to express the motion of an object using narrative, mathematical, and graphical representations.
- **MA Science and Technology/Engineering Standards (2016)**
  - HS-PS2-10(MA). Use free-body force diagrams, algebraic expressions, and Newton’s laws of motion to predict changes to velocity and acceleration for an object moving in one dimension in various situations.
Misconceptions Addressed

- Introductory physics students often confuse position and velocity. This lesson helps delineate the difference between the two. (Question 4d)
- Further information about student misconceptions on this topic can be found [here](#).

Primary Sources

- Bite “Quantum Conspiracy Theory” based on:

- Misconceptions

Materials

Copies of the Student Handout and Science Bite for each student

Time

This lesson should take approximately one 50-minute class period.

Student Prior Knowledge

Students should know how to find velocity in one dimension given distance and time. They should have been introduced to the idea of negative velocity. Additionally, students should know what a frame of reference is and how to calculate the velocity of an object in a moving frame of reference.

Instructions and Teacher Tips

- General Procedure
  - Hand students the student document and have them read the introduction and answer the first two analysis questions.
  - Review the answers to the first two analysis questions to make sure students have the background information necessary to get the most out of the Science Bite.
  - Have students read the Science Bite and complete questions 3–7.
  - Review the answers to questions 3–7 to check for student understanding.

- Tips, Extensions, and Variations
  - One option to launch and wrap-up this lesson is to have students fill out a Know, Want to Know, Learned (KWL) chart. At the beginning of the lesson, students can list the things they know or have heard about Heisenberg’s uncertainty principle or quantum physics more generally and the things they want to know about either. Have students share out some of the things they know and want to know. At the end of the lesson, have...
students reflect on and write down what they have learned.

- As an extension for more advanced students, you could give students positions for the different vehicles and have them calculate when the vehicles will pass each other. In order to introduce different representations of motion, you could also have students graph the position of the vehicles over time and represent their motion with dot diagrams.

- To get students to think more about uncertainty, you could discuss the uncertainty in the position of the particles in question 3. Although we can estimate where the particles are on the coordinate plane, we can’t say we know their exact location as we are limited by the scale of the coordinate system and the fact that the particles themselves are pretty large, so what are we choosing to use as their position (midpoint, top edge, right edge, etc.).

**Background Information and Research Details**

- The mathematics of Heisenberg’s uncertainty principle is generally taught to upper level physics undergraduate students in a quantum mechanics course, but the concept of the principle can be understood without a significant amount of physics background.

- Quantum mechanics is often viewed by students as both an extremely interesting and exciting area of science as well as one of science’s most challenging and confusing domains. Many professional scientists feel this way too! The hallmark of quantum effects are that they are different than and seem to violate the laws of “classical” physics that govern our everyday life. Nobel prize-winning physicist Richard Feynman famously wrote, “I think I can safely say that nobody understands quantum mechanics.”

- Heisenberg’s uncertainty principle is a foundational concept in physics and chemistry because it governs the structure of atoms. Electrons in atoms exist as “clouds,” regions of space around atomic nuclei where they have a higher or lower probability to appear, because the exact position of the electron cannot be known at any specific time due to the uncertainty principle. The shape of these probability clouds is largely responsible for determining how atoms and molecules interact with each other to form chemical bonds and reactions.

- An important implication of the uncertainty principle is that the mere act of observation has consequences on the properties of the quantum world. If we increase our certainty in the position of a particle, our uncertainty about its momentum must grow. The famous Schrödinger’s Cat thought experiment, in which the well being of an obscured animal is used as an analogy for the state of an unobserved quantum particle, depicts the consequences of observation in very vivid terms.

- This article’s premise is related to a well known thought experiment performed in 1935 by Albert Einstein, Boris Podolsky, and Nathan Rosen (known as the EPR Paradox), which poses that two particles can be quantum mechanically “entangled” such that measuring a property of one particle would have consequences for the other particle. This must occur because the act of measurement changes our uncertainty, and if the two particles have quantum entanglement, then the act of measurement changes the uncertainties of both particles. In Einstein’s thinking, for this kind of change to be propagated between the two
particles instantly, particularly faster than the speed of light, constituted “spooky action at a distance.” Scientists have generated a better understanding of how this entanglement happens over time, and indeed today it is the basis of technologies such as quantum computing and cryptography.

In the research article, the scientists discuss a laboratory experiment performed with magnetic oscillators that they use to support the theory they present.

Although we make the analogy between positive/negative mass and positive/negative charge, scientists have theorized some key differences. While positive charges will attract negative charges and repel positive charges, in theory, positive mass will attract all mass, positive or negative and negative mass will repel all masses, positive or negative. This may seem contradictory, but a positive and negative mass will both repel and attract each other, both forces will exist.

Big Question Discussion

This lesson addresses the Big Question “What does it mean to observe?” In particular students should come away with an appreciation of how difficult it is for scientists to make observations of atoms and subatomic particles. If you choose to delve into the Big Question, consider the following ideas:

Have a wrap-up discussion around how there are many things that scientists can’t observe with the naked eye. Often these observations seem contrary to our everyday experience. Does that make these observations invalid? Does that make our everyday experience invalid? How can we reconcile these two types of observations?

Have a discussion of scale with students—a grain of sand contains approximately $1 \times 10^{18}$ atoms, consider how small an atom must be for that many atoms to be contained in a grain of sand. Why do you think it is so hard for scientists to make observations at that size?

Answers

1. Compare and contrast classical physics with quantum physics.

Classical physics is the study of our larger world. It helps describe the motion of large objects that we can in general see with our naked eye. Quantum physics is the study of things that are super small and that we’ll never be able to see with the naked eye. They behave very differently than the large objects in our everyday lives. Both classical physics and quantum physics help us explain the world around us.

2. Why do you think most first year physics courses focus on classical physics instead of quantum physics?

Classical physics is more relatable than quantum physics. You can use your everyday experience to help you explain concepts in classical physics.

3. Examine the diagram below.

Exploring Position and Momentum in the Quantum World
a. What is the position of point A with respect to the origin? (4 m, 3 m)

b. What is the position of point B with respect to the origin? (15 m, 9 m)

c. What is the position of point B with respect to point A? (11 m, 6 m)

d. How is finding the position of B with respect to A similar to how the researchers propose using a negative mass particle to measure the trajectory of a positive mass particle? How is it different?

The researchers propose finding the trajectory of a particle by finding its trajectory relative to a negative mass particle. We found the position of particle B relative to position A. We could imagine particle A as our negative mass particle and particle B as our positive mass particle. We found position instead of trajectory. Trajectory implies the object was moving, ours were stationary objects on a coordinate plane.

4. A person is standing on city sidewalk. A blue car and a orange truck on the street, as shown in Figure 1. Both vehicles are traveling in the positive direction. The blue car travels 60 meters in 4 seconds. The orange truck travels 36 meters in 3 seconds.

Figure 1.

a. What is the velocity of the blue car with respect to the person on the sidewalk?

\[ v = \frac{\Delta x}{\Delta t} = \frac{60 \text{ m}}{4 \text{ s}} = 15 \text{ m/s} \]

b. What is the velocity of the orange truck with respect to the person on the sidewalk?

\[ v = \frac{\Delta x}{\Delta t} = \frac{36 \text{ m}}{3 \text{ s}} = 12 \text{ m/s} \]

c. What is the velocity of the blue car with respect to the orange truck?

\[ v_2 - v_1 = 15 \text{ m/s} - 12 \text{ m/s} = 3 \text{ m/s} \]

d. How is finding relative position similar to finding relative velocity for two objects moving in the same direction? More generally, how are position and velocity related?

Finding relative position was similar to finding relative velocity, because for both you just need to find the difference between the two points or velocities while being mindful of the direction. Position is related to velocity in that velocity is the rate of change of position.

5. Now the person on the sidewalk sees a purple car going in the negative direction and a yellow van traveling in the positive direction. The purple car travels 16 meters in 2 seconds, while the yellow van travels 11 meters in 2.5 seconds.
a. What is the velocity of the purple car with respect to the person on the sidewalk?

\[ v = \frac{\Delta x}{\Delta t} = \frac{-16 \text{ m}}{2 \text{ s}} = -8 \text{ m/s} \]

b. What is the velocity of the yellow van with respect to the person on the sidewalk?

\[ v = \frac{\Delta x}{\Delta t} = \frac{11 \text{ m}}{2.5 \text{ s}} = 4.4 \text{ m/s} \]

c. What is the velocity of the purple car with respect to the yellow van?

\[ v_2 - v_1 = 4.4 \text{ m/s} - (-8 \text{ m/s}) = 4.4 \text{ m/s} + 8 \text{ m/s} = 12.4 \text{ m/s} \]

6. Consider how we found the velocity of the blue car with respect to the orange truck and the velocity of the purple car with respect to the yellow van.

a. How is that similar to how the researchers propose using a negative mass particle to measure the trajectory of a positive mass particle?

The researchers were proposing finding the trajectory of the positive mass particle using a negative mass particle as their reference frame. By finding the trajectory they were looking at tracking how the position of the particle changes over time. This is similar to looking at the velocity of a car with respect to another car, because you are thinking about how the position of one car changes over time with respect to the second car.

b. In what ways do both of our analogies (comparing the research to finding relative position and finding relative velocity) not accurately describe the research?

The researchers look at trajectory, which is a combination of velocity and position, not just one or the other. Additionally, all of our examples involve only objects with positive mass. Our car examples dealt with the velocity of our “particles” while the research focused on the momentum of the particles.

7. Connect to the Big Question. This research deals with particles that are far too small for human eyes to see, yet asking how we can make observations about their position and momentum gives us new insights into the fundamental properties of the universe. How do these scientists' proposed observations differ from the observations you make in your everyday life? Does it change your personal definition of what it means to observe? Explain.
Student answers may vary. Sample response: The scientists are proposing things that we may never be able to observe. Their work is theoretical, while our everyday experience is grounded in what we observing happening. We have our senses to guide us, while these scientists are thinking of things that we can only visualize using diagrams and models. We will never be able to see them ourselves. It does not change my personal definition of what it means to observe, because I think observations are more than just what we can see, hear, touch, etc. Learning about these proposed observations just added a new dimension to what I already thought.